One Sample t-test: Special Case:

Matched Pairs Design – 2 trts. – every subject gets both trts.

2 sets of data
– linked
* compare
→ 1 set of data

- How many samples?

single sample procedure

- What type of design?

comparative design

- Most common types of problems:
  - 1st group / 2nd group

* Before/After measurements on same exp units

    - always do after – before

- Can also be…

2 subjects matched up

Ex: twins
married couples
* not the best design b/c lurking variables.

- Two sets of data must be…

dependent on each other

- How do we compare the 2 sets of data?
    - look @ differences b/w.

- Once we have the differences…
    - We use… one sample t test/interval
    - Test… the (mean) avg. difference
    - How do we write this in the hypotheses?

$H_0: \mu_d = \underline{\#}$ (often # is 0)

$H_a: \mu_d \gtrless\neq \underline{\#}$

- Conclusion:
  - -reject/fail to reject...
  - We have suff. evid. that the mean (avg.) difference of/btw. ______ is...
- Assumptions:
  - SRS
  - normal pop. of differences

    or
  - $n_d \geq 30$

Example #1:

The SAT prep course here at CB south claims to increase the SAT math scores of its students by 30 points. We don't think it is this much. We think it is less. We measure 5 students' SAT math scores before and after taking the class and find the following data. Test the hypotheses.

L_1 L_2

Subject	Before	After
A	500	520
B	430	440
C	490	480
D	550	590
E	520	550

Differences $= L_3 = L_2 - L_1$

$H_0: \mu_d = 30$

$H_a: \mu_d < 30$

$t = \frac{\bar{x} - \mu}{s/\sqrt{n}} = -1.395$

$P(t < -1.395 \mid df = 4) = 0.1177$

- We fail to reject...
- Have suff. evid. that the avg. diff. btw. scores before & after SAT prep course is 30 pts.

Example #2:

We want to test the differences between Mrs. Tannery's (CB East teacher) Block 2 and Block 4 SAT prep classes. Mrs. Tannery thinks that she taught Block 2 better than Block 4. Using the data below, test the hypotheses. The data gives the average class score for each of the 9 weeks of the course.

Date	Block 2 (L_1)	Block 4 (L_2)
10-Sep	480	472
17-Sep	497	495
24-Sep	505	502
1-Oct	499	524
8-Oct	517	515
15-Oct	524	530
22-Oct	552	531
29-Oct	540	531
5-Nov	583	574

$L_3 = L_2 - L_1$

$H_0: \mu_d = 0$

$H_a: \mu_d < 0$

$L_3 = L_1 - L_2$

$H_0: \mu_d = 0$

$H_a: \mu_d > 0$

$$t = \frac{\bar{x}_d - \mu_d}{s_d / \sqrt{n_d}} = -0.605$$

$$P(t < -0.605 \mid df = 8) = 0.2811$$

- We fail to reject...
- We have suff. evid. that the avg. diff btw. Blk 2 & 4 SAT scores is equal to 0 pts. She teaches both classes the same.

Try the worksheet problems on your own

Wksht

#1 a) $H_0: \mu_d = 0$

$H_a: \mu_d \neq 0$

b) $t = \dfrac{\bar{x}_d - \mu_d}{s_d / \sqrt{n_d}} = \boxed{12.8233}$

$2 \cdot P(t > 12.8233 \mid df = 4) = \boxed{0.0002}$

We reject Ho b/c p-value < $\alpha = 0.05$.

We have suff. evid. that the avg. diff. of tire wear btw. tires A & B is not equal to 0. So there is a diff. btw. the wear on tires A & B.

c) $\overline{x}_d \pm t^* \left(\frac{s_d}{\sqrt{n_d}} \right) = (0.40, 0.56)$

We are 90% conf. that the avg. diff. in wear on tires A and B is btw. 0.40 and 0.56 units.

2 means

- response in 2 distinct groups
- a sample from a distinct pop.
- independent of other group

2 sample T-test

1) Assump.
2) Hyp.
3) Test Stat
4) P-value
5) Conclusion

Pop 1	Pop 2
μ_1	μ_2
σ_1	σ_2
n_1	n_2
$\bar{x}_1$	$\bar{x}_2$
s_1	s_2

Hypotheses

- comparing 2 means

* $H_0: \mu_1 = \mu_2$

$H_a: \mu_1 \underset{\neq}{\gtrless} \mu_2$

OR

$H_0: \mu_1 - \mu_2 = 0$

$H_a: \mu_1 - \mu_2 \underset{\neq}{\gtrless} 0$

(* no #'s in hyp)

Test Stat

$$t = \frac{\text{stat.} - \text{param}}{\text{std. dev of stat}} = \boxed{\frac{(\bar{x}_1 - \bar{x}_2)}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}}$$

$P(t \geq \underline{\text{test stat}} \mid df= \quad) =$

- df:
  - smaller of $n_1 - 1$ or $n_2 - 1$

    OR

  - * calculator (not whole #)

Concl

- same reject / fail to reject...
- suff evid. that the mean of __#1__ is $\underline{\geq \neq}$ the mean of __#2__.

2-Samp T-test Pooled: $\boxed{\text{NO}}$

Conf Int:

Formula

statistic $\pm$ (crit. val)(std. dev. of stat.)

$$\underbrace{(\bar{x}_1 - \bar{x}_2)}_{\text{estimate}} \pm \underbrace{(t^*)\left(\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}\right)}_{\text{moe}} \qquad \mu_1 - \mu_2$$

df = same as 2 samp t-test (calc.)

Concl.:

We are ___% conf. that the diff btw. the means of ___#1___ and ___#2___ is btw ___ and ___ units.

2 Samp T-Interval

Assump

- 2 independent SRS
- 2 normal pop.

  or

  n_1
  $n_2 \geq 30$

Ex:

$H_0: \mu_T = \mu_C$

$H_a: \mu_T > \mu_C$

$$t = \frac{\bar{X}_1 - \bar{X}_2}{\sqrt{\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}}} = 1.49$$

$$P(t > 1.49 \mid df = 41.49) = 0.07$$

- fail to reject...
- suff. evid. that the avg scores of control & trt. are equal.

$$(\bar{X}_T - \bar{X}_C) \pm t^* \cdot \sqrt{\frac{S_T^2}{n_T} + \frac{S_C^2}{n_C}}$$

We are 95% conf. that the difference btw. trt & control scores is btw. -1.277 & 8.3.. pts.